平面磁流体波穿透圆柱面分层赤道电离层和大气层的理论

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摘　要　给出了平面磁流体快波穿透圆柱面分层模型的赤道电离层和大气层的理论. 平面磁流体快波被分解成柱面波, 柱面波在圆柱分层介质中的传播被化为一个两点边值问题, 给出了所需统治方程和边界条件.

关键词　磁流体波; 电离层; 分层介质

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Theory of Penetration of Hydromagnetic Wave into Cylindrical Model of Equatorial Ionoshere

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Abstract　This paper presents a theory of penetration of fast mode hydromagnetic plane wave into the cylindrically stratified equatorial ionosphere and atmosphere. The fast mode hydromagnetic plane wave is decomposed into cylindrical wave modes. Then the propagation of cylindrical wave modes in cylindrically stratified ionosphere and atmosphere are solved as a two-point boundary value problem. The governing equations are from Maxwell equations. They are a system of four complex first order differential equations. The lower boundary of the solved region is the ground.The upper boundary is the plane of 500 km altitude. Above the upper boundary, the wave electric field component along the magnetic field is zero. The wave magnetic field component along the ambient magnetic field satisfy the Bessel equation. Therefore the wave magnetic field component can be decomposed into cylindrical Bessel functions. The wave magnetic field component at the upper boundary are formed by the summation of inward going wave component and outward going wave component. The wave magnetic field is differentiated with respect to r. Then the inward going wave component is expressed in terms of the Bessel function solution and its differentiation. Through the continuation of the electric and magnetic field along the boundary, the boundary conditions are therefore formed. The cylindrical wave modes are composed to give the spatial distribution of the electromagnetic wave fields.

Key words　Hydromagnetic wave, Ionosphere, Layered medium

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1 引言

磁层的磁流体波可以穿透电离层和大气层而到达地面. 对于平面分层介质的情形, Zhang[1] 首先提出了用两点边值问题的方法解电磁波传播的问题, Hughes 和 Southwood[2] 与 Zhang 和 Cole[3] 计算了平面磁流体波穿透水平分层的高纬度电离层和大气层的电磁波传播情况, Zhang 和 Cole[4],[5] 计算了圆柱对称的磁流体波穿透水平分层的高纬度电离层和大气层的情况, 并计算了平面磁流体波穿透水平分层的赤道电离层和大气层的电磁波传播情况, 本文给出了平面磁流体快波穿透圆柱面分层的赤道电离层和大气层的理论.

2 物理模型与统治方程

如图 1 所示, 假定赤道电离层和大气层是圆柱面分层的, 即沿南北方向是无限长且均匀的, 在圆柱坐标系 (r,φ,z) 中, r 垂直向上, φ 指向东, z 指向北, 地球磁场 $\boldsymbol{B}_0$ 指向 z 也即指向北且假定是均匀的. 磁层中的一列平面磁流体快波向赤道电离层传播, 并进入电离层和大气层, 本文要求 $R_0 \leqslant r \leqslant R_1$ 区域中的电磁场, R_0 是区域的下边界即地面, R_1 是区域的上边界, 例如 500 km 的高度, 之所以提快模式的平面磁流体波是因为快模式的磁流体波的传播方向是各向同性的, 而 Alfven 波是沿磁力线传波的.

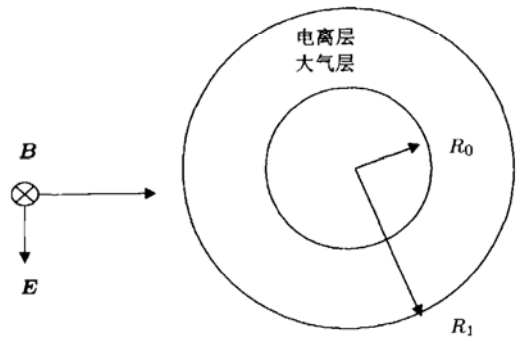


图 1 问题的几何描述

Fig.1. Geometry of the problem

设电场和磁场随时间的变化为 $\mathrm{e}^{\mathrm{i}\omega t}$, 则 Maxwell 旋度方程为

$$\nabla \times \boldsymbol{E} = -\mathrm{i}\omega \boldsymbol{B}, \tag{1}$$

$$\nabla \times \boldsymbol{B} = \mathrm{i}\frac{\omega}{c^2}\boldsymbol{\varepsilon} \cdot \boldsymbol{E}, \tag{2}$$

其中, c 为真空中的光速, $\boldsymbol{\varepsilon}$ 为介电张量, 其表达式为

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_1 & -\varepsilon_2 & 0 \\ \varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix}, \tag{3}$$

介电张量 $\boldsymbol{\varepsilon}$ 与电导率张量 $\boldsymbol{\sigma}$ 以如下公式联系:

$$\boldsymbol{\varepsilon} = \boldsymbol{I} - \frac{\mathrm{i}}{\varepsilon_0 \omega}\boldsymbol{\sigma}, \tag{4}$$

其中, ε_0 为真空介电常数, 电导率 $\boldsymbol{\sigma}$ 只随高度变化.

设磁层中的平面磁流体快波的磁场分量沿 z 方向极化, 且沿 z 方向的变化为 $\mathrm{e}^{-\mathrm{i}k_z z}$, 它可被分解为如下柱面波 [6] 形式:

$$\begin{aligned} B_z &= \mathrm{e}^{-\mathrm{i}(k_r x + k_z z)} \\ &= \mathrm{e}^{-\mathrm{i}k_z z} \sum_{n=-\infty}^{\infty} \mathrm{i}^{-n} J_n(k_r r)\mathrm{e}^{\mathrm{i}n\varphi}, \end{aligned} \tag{5}$$

其中, $J_n(k_r r)$ 为 n 阶贝塞尔函数且

$$k_r^2 + k_z^2 = k_{\mathrm{A}}^2 = \frac{\omega^2}{v_{\mathrm{A}}^2}, \tag{6}$$

其中, v_{A} 为 Alfven 速度, 其表达式为

$$v_{\mathrm{A}} = \sqrt{\frac{B_0^2}{\mu_0 N_0 m_{\mathrm{i}}}} = \sqrt{\frac{B_0^2}{\mu_0 \rho}}, \tag{7}$$

其中, μ_0 为真空磁导率, N_0 为粒子数密度, m_{i} 为离子质量, ρ 为质量密度, 因子 $\mathrm{e}^{-\mathrm{i}k_z z}$ 的引入是因为冬夏季度 k_z 可不为零, 春秋季度 k_z 可为零.

对求解区域 $R_0 \leqslant r \leqslant R_1$, 设场量的变化为 $\mathrm{e}^{\mathrm{i}(\omega t + n\varphi - k_z z)}$, 即电磁场的第 n 个谐波分量为

$$\begin{aligned} \boldsymbol{E}_n(r,\varphi,z) = &\big[\boldsymbol{e}_r E_{nr}(r) + \boldsymbol{e}_\varphi E_{n\varphi}(r) + \\ &\boldsymbol{e}_z E_{nz}(r)\big]\mathrm{e}^{\mathrm{i}(n\varphi - k_z z)}, \end{aligned} \tag{8}$$

$$\begin{aligned} \boldsymbol{B}_n(r,\varphi,z) = &\big[\boldsymbol{e}_r B_{nr}(r) + \boldsymbol{e}_\varphi B_{n\varphi}(r) + \\ &\boldsymbol{e}_z B_{nz}(r)\big]\mathrm{e}^{\mathrm{i}(n\varphi - k_z z)}. \end{aligned} \tag{9}$$

代入式 (1) 和 (2) 得到场的第 n 个谐波分量满足下列常微分方程组:

$$\begin{aligned} \frac{\mathrm{d}E_{n\varphi}}{\mathrm{d}r} = &\Big(\frac{\mathrm{i}n\varepsilon_2}{\varepsilon_1 r} - 1\Big)E_{n\varphi} + \frac{\mathrm{i}nc^2 k_z}{\omega\varepsilon_1 r}B_{n\varphi} + \\ &\mathrm{i}\Big(\frac{c^2 n^2}{\omega\varepsilon_1 r^2} - \omega\Big)B_{nz}, \end{aligned} \tag{10}$$

$$\frac{\mathrm{d}E_{nz}}{\mathrm{d}r} = -\frac{\mathrm{i}k_z\varepsilon_2}{\varepsilon_1}E_{n\varphi} + \mathrm{i}\Big(\omega - \frac{c^2k_z^2}{\omega\varepsilon_1}\Big)B_{n\varphi} - \frac{\mathrm{i}c^2nk_z}{\omega\varepsilon_1 r}B_{nz}, \tag{11}$$

$$\frac{\mathrm{d}B_{n\varphi}}{\mathrm{d}r} = -\frac{\mathrm{i}nk_z}{\omega r}E_{n\varphi} + \mathrm{i}\Big(\frac{\omega\varepsilon_3}{c^2} - \frac{n^2}{\omega r^2}\Big)E_{nz} - B_{n\varphi}, \tag{12}$$

$$\frac{\mathrm{d}B_{nz}}{\mathrm{d}r} = \mathrm{i}\Big(\frac{k_z^2}{\omega} - \frac{\omega\varepsilon_2^2}{c^2\varepsilon_1} - \frac{\omega\varepsilon_1}{c^2}\Big)E_{n\varphi} + \frac{\mathrm{i}nk_z}{\omega r}E_{nz} - \frac{\mathrm{i}\omega\varepsilon_2c^2k_z}{\omega c^2\varepsilon_1}B_{n\varphi} - \frac{\mathrm{i}\omega\varepsilon_2 n}{\omega\varepsilon_1 r}B_{nz}, \tag{13}$$

$$E_{nr} = \frac{\varepsilon_2}{\varepsilon_1}E_{n\varphi} + \frac{c^2k_z}{\omega\varepsilon_1}B_{n\varphi} + \frac{c^2n}{\omega\varepsilon_1 r}B_{nz}, \tag{14}$$

$$B_{nr} = -\frac{k_z}{\omega}E_{n\varphi} - \frac{n}{\omega r}E_{nz}. \tag{15}$$

方程组 (10)—(13) 可用两点边值问题求解，为此需要边界条件.

3 边值关系

在下边界，即地表面处 $r = R_0$, 因大地可看成导体，所以下边界处可有如下边界条件：

$$E_\varphi = 0, \quad r = R_0, \tag{16}$$

$$E_z = 0, \quad r = R_0. \tag{17}$$

在上边界 $r = R_1$ 以外的区域，气体稀薄，磁流体波可以用以下的单流体的理想 MHD 方程和 Maxwell 旋度方程描述 [7]:

$$\rho\frac{\partial \boldsymbol{v}}{\partial t} = \boldsymbol{j}\times\boldsymbol{B}_0, \tag{18}$$

$$\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}_0 = 0, \tag{19}$$

$$\nabla\times\boldsymbol{B} = \mu_0\boldsymbol{j}, \tag{20}$$

$$\nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t}. \tag{21}$$

其中，ρ 为等离子体质量密度，$\boldsymbol{B}_0$ 为地球背景磁场，$\boldsymbol{v},\boldsymbol{j},\boldsymbol{E},\boldsymbol{B}$ 为随时间变化的波传播量. $\boldsymbol{B}$ 比 $\boldsymbol{B}_0$ 小很多. 在 $r > R_1$ 的区域假定 ρ 和 $\boldsymbol{B}_0$ 是均匀的. $\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}_0 = 0$ 是零电阻等离子体的欧姆定律的简化形式，在 $\omega < \omega_\mathrm{i}$ 的情况下成立，式 (18)—(19) 为冷等离子体近似，其中压力梯度被忽略了. 位移电流在频率很低的情况下被忽略了. 在圆柱坐标下，当场量的变化为 $\mathrm{e}^{\mathrm{i}(n\varphi - k_z z + \omega t)}$ 时，从式 (18)—(21) 可得到 [7]

$$E_{nz} = 0, \quad r \geqslant R_1, \tag{22}$$

$$\frac{\mathrm{d}^2B_{nz}}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}B_{nz}}{\mathrm{d}r} + \Big(k_r^2 - \frac{n^2}{r^2}\Big)B_{nz} = 0, \tag{23}$$

其中，

$$k_r^2 = \frac{\omega^2}{v_\mathrm{A}^2} - k_z^2, \tag{24}$$

因为 B_{nz} 满足式 (23) 贝塞尔方程，所以场量 B_{nz} 可由入射波和散射波组成，其中入射波分量 B_{nz}^i 和散射波分量 B_{nz}^s 可表示如下 [8]:

$$B_{nz}^\mathrm{i} = J_n(k_r r), \tag{25}$$

$$B_{nz}^\mathrm{s} = A_n H_n^{(2)}(k_r r), \tag{26}$$

其中，A_n 为散射系数，$H_n^{(2)}(k_r r)$ 为第二类 n 阶汉克尔函数，即

$$B_{nz} = B_{nz}^\mathrm{i} + B_{nz}^\mathrm{s} = J_n(k_r r) + A_n H_n^{(2)}(k_r r). \tag{27}$$

则有

$$\frac{\mathrm{d}B_{nz}}{\mathrm{d}r} = k_r J_n'(k_r r) + k_r A_n H_n^{(2)'}(k_r r) = k_r\frac{J_n'(k_r r)}{J_n(k_r r)}B_{nz}^\mathrm{i} + k_r\frac{H_n^{(2)'}(k_r r)}{H_n^{(2)}(k_r r)}B_{nz}^\mathrm{s}, \tag{28}$$

由式 (27) 和 (28) 消去 B_{nz}^s 得到

$$\frac{H_n^{(2)'}(k_r r)}{H_n^{(2)}(k_r r)}B_{nz} - \frac{1}{k_r}\frac{\mathrm{d}B_{nz}}{\mathrm{d}r} = \Big[\frac{J_n'(k_r r)}{J_n(k_r r)} - \frac{H_n^{(2)'}(k_r r)}{H_n^{(2)}(k_r r)}\Big]B_{nz}^\mathrm{i}, \tag{29}$$

其中，$J_n'(k_r r)$ 和 $H_n^{(2)'}(k_r r)$ 表示对宗量求导数，将式 (13) 和 (25) 代入 (29) 并令 $r = R_1$ 则得到上边界的一个边界条件如下：

$$-\frac{\mathrm{i}}{k_r}\Big(\frac{k_z^2}{\omega} - \frac{\omega\varepsilon_2^2}{c^2\varepsilon_1} - \frac{\omega\varepsilon_1}{c^2}\Big)E_{n\varphi} - \frac{\mathrm{i}k_z n}{k_r\omega R_1}E_{nz} + \frac{\mathrm{i}\omega\varepsilon_2c^2k_z}{\omega c^2\varepsilon_1 k_r}B_{n\varphi} + \Big[\frac{H_n^{(2)'}(k_r R_1)}{H_n^{(2)}(k_r R_1)} + \frac{\mathrm{i}\omega\varepsilon_2 n}{\omega\varepsilon_1 R_1 k_r}\Big]B_{nz} = \Big[\frac{J_n'(k_r R_1)}{J_n(k_r R_1)} - \frac{H^{(2)'}(k_r R_1)}{H_n^{(2)}(k_r R_1)}\Big]J_n(k_r R_1), \tag{30}$$

上边界的另外一个边界条件是式 (22).

方程 (10)—(13) 以及边值关系式 (16)—(17) 和式 (22) 与 (30) 组成了求解 $E_{n\varphi}(r), E_{nz}(r), B_{n\varphi}(r), B_{nz}(r)$ 的两点边值问题，目前有很多现成的计算机程序计算两点边值问题. 由式 (14) 和 (15) 可求得 $E_{nr}(r)$ 和 $B_{nr}(r)$, 这些量求出以后，可由如下表达式得出电离层和大气层的电磁场空间分布:

$$E_r(r,\varphi,z) = \sum_{n=-\infty}^{\infty} \mathrm{i}^{-n} E_{nr}(r) \mathrm{e}^{\mathrm{i}(n\varphi - k_z z)}, \tag{31}$$

$$E_\varphi(r,\varphi,z) = \sum_{n=-\infty}^{\infty} \mathrm{i}^{-n} E_{n\varphi}(r) \mathrm{e}^{\mathrm{i}(n\varphi - k_z z)}, \tag{32}$$

$$E_z(r,\varphi,z) = \sum_{n=-\infty}^{\infty} \mathrm{i}^{-n} E_{nz}(r) \mathrm{e}^{\mathrm{i}(n\varphi - k_z z)}, \tag{33}$$

$$B_r(r,\varphi,z) = \sum_{n=-\infty}^{\infty} \mathrm{i}^{-n} B_{nr}(r) \mathrm{e}^{\mathrm{i}(n\varphi - k_z z)}, \tag{34}$$

$$B_\varphi(r,\varphi,z) = \sum_{n=-\infty}^{\infty} \mathrm{i}^{-n} B_{n\varphi}(r) \mathrm{e}^{\mathrm{i}(n\varphi - k_z z)}, \tag{35}$$

$$B_z(r,\varphi,z) = \sum_{n=-\infty}^{\infty} \mathrm{i}^{-n} B_{nz}(r) \mathrm{e}^{\mathrm{i}(n\varphi - k_z z)}, \tag{36}$$

4 结语

通过以上分析，给出了平面磁流体快波穿透圆柱面分层模型的赤道电离层和大气层的理论. 平面磁流体快波被分解成柱面波，柱面波在圆柱分层介质中的传播被化为一个两点边值问题，给出了所需的统治方程和边界条件.

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