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空间运动体上梁的三维动力学建模和仿真*

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摘 要 对附着在空间运动体上梁的动力学问题进行了研究，运用 Lagrange 方法建立了中心刚体作任意三维运动时，梁作横向振动和纵向振动的动力学方程，考虑了柔性梁的横向弯曲变形所带来的纵向位移，这种耦合变形使所得到的振动方程包含了动力刚化效应，这是与传统的梁振动模型的不同之处，最后通过仿真算例验证了该方法.

关键词 柔性梁；动力刚化；三维振动

中图法分类号 O 313.7

Three-Dimensional Dynamic Modeling and Simulation of a Beam Attached to a Spatially Moving Base

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Abstract The three-dimensional dynamics of a flexible cantilever beam attached to a moving rigid body undergoing an arbitrarily three-dimensional large overall motion is investigated in this paper. A set of dynamic equations for two-dimensional transverse and one-dimensional longitudinal vibrations of the flexible beam is established by using Lagrange's dynamic method. In the construction of the governing equations of motions the coupling effects of the so called transverse deformation-induced longitudinal deformation is included, which leads to the consideration of the dynamic stiffening effects in the obtained dynamic equations. An example is given to show the validity of the method presented in this paper and also the significant effects of the dynamic stiffening terms on the deformation and the dynamic characteristic of the flexible beam, and the difference between the present modeling theory and the traditional vibration theory as well. The traditional vibration theory of flexible beam will produce large error when the flexible beam itself is at higher speed large overall motion. Once the speed reaches at a critical value, the traditional dynamic system will diverge. Whereas the present model has high precision and the dynamic system converges even if the flexible beam undergoes higher speed large overall motion.

Key words Flexible beam, Dynamic stiffening, Three-dimensional vibrations

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1 引言

运动体上柔性梁的动力学行为与固定基座上的柔性梁的动力学行为有本质上的差别，这种差别主要来自于运动体的运动对柔性梁变形运动的影响. 人们在对柔性机械臂、带柔性附件的航天器进行动力学分析时提出了运动体 (中心刚体)—— 柔性梁这样一类系统. 1987 年, Kane[1] 对运动基上柔性梁系统动力学问题进行了精确的表述和研究，指出在大范围刚体运动作高速旋转时传统的耦合建模方法 [2] 得到的弹性梁的变形将无限增大，此结果与实际情况相反. 并首次提出 “动力刚化” 概念，即柔性梁的大位移旋转运动将引起其刚度的增加，这与传统的结论刚好相反. 自 Kane 提出 “动力刚化” 概念以来，学术界给予了极大的关注 [3]~[14]. 然而在以往的这些研究中，给出的仿真例子大多限于平面运动，很少涉及三维空间运动体上梁的三维动力学问题. 本文对附着在三维空间运动体上的柔性梁的三维振动问题进行了研究，运用 Lagrange 方法建立了运动体作任意三维运动时，梁作横向二维振动和纵向一维振动的动力学方程，此动力学方程考虑了柔性梁的横向变形所带来的纵向位移，计及 “动力刚化” 效应. 本文结论既区别于传统的梁三维动力学理论，又较以往的仅考虑梁平面运动的 “动力刚化” 模型前进了一步，并且通过设定空间运动体的运动模式，本文所得到的动力学模型可以退化到中心刚体-外梁、中心刚体-内梁和中心刚体-纵向梁等情形. 另外，本文给出的是一种程式化的建模法，这为 “动力刚化项” 程式化地引入柔性多体系统动力学理论作了可行性预研.

2 系统描述

考虑如图 1 所示的系统，E-$\xi\eta\zeta$ 为惯性坐标系，O-xyz 固结在运动刚体上，作为描述梁变形的浮动坐标系，柔性梁以悬臂方式固结在中心刚体 O 点处，并在未变形时梁的自然位置沿 Ox 轴方向，梁长度为 L，考虑梁 x, y, z 方向的变形运动. 设浮动坐标系 O-xyz 的运动已知，即给定角速度 $\boldsymbol{\omega}_0$，角加速度 $\boldsymbol{\varepsilon}_0$，原点 O 的速度 $\boldsymbol{v}_0$ 和加速度 $\boldsymbol{a}_0$. 图中 $\boldsymbol{r} = x\boldsymbol{i}$.

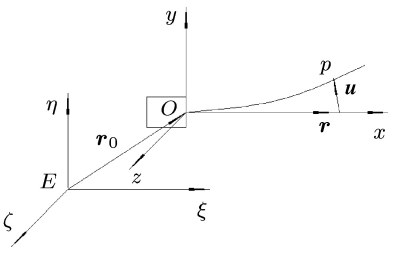


图 1 刚柔耦合系统

Fig.1 Rigid-flexible coupling system

3 动力学方程

3.1 柔性梁的动能

柔性梁中心轴线处距 O 点为 x 处的点的速度为

$$\boldsymbol{v} = \boldsymbol{v}_0 + \boldsymbol{\omega}_0 \times (\boldsymbol{r} + \boldsymbol{u}) + \mathring{\boldsymbol{u}}. \tag{1}$$

其中, $(\circ)$ 表示矢量在运动刚体坐标系 O-xyz 中求导，即求相对导数. $\boldsymbol{v}$ 在运动刚体坐标系 O-xyz 中表达为

$$\begin{aligned}\boldsymbol{v} =&(v_{0x} - \omega_{0z}u_y + \omega_{oy}u_z + \dot{u}_x)\boldsymbol{i}+\\ &[v_{0y} + \omega_{0z}(x + u_x) - \omega_{ox}u_z + \dot{u}_y]\boldsymbol{j}+\\ &[v_{0z} - \omega_{0y}(x + u_x) + \omega_{ox}u_y + \dot{u}_z]\boldsymbol{k}.\end{aligned} \tag{2}$$

其中, u_x, u_y, u_z 分别为弹性变形矢量 $\boldsymbol{u}$ 在 x, y, z 轴上的投影, $v_{0x}, v_{0y}, v_{0z}, \omega_{0x}, \omega_{0y}, \omega_{0z}$ 分别为坐标系 O-xyz 的速度 $\boldsymbol{v}_0$，角速度 $\boldsymbol{\omega}_0$ 在运动刚体坐标系 O-xyz 上的投影. 柔性梁的动能为

$$\begin{aligned}T =&\frac{1}{2}\rho A\int_0^L \{[v_{0x} - \omega_{0z}u_y + \omega_{oy}u_z + \dot{u}_x]^2+\\ &[v_{0y} + \omega_{0z}(x + u_x) - \omega_{ox}u_z + \dot{u}_y]^2+\\ &[v_{0z} - \omega_{0y}(x + u_x) + \omega_{ox}u_y + \dot{u}_z]^2\}\mathrm{d}x.\end{aligned} \tag{3}$$

式中, ρ 为梁的质量密度, A 为梁横截面积, ρA 即为梁单位长度的质量.

3.2 柔性梁的势能

在高速旋转时，重力的影响往往不重要，为简便, 忽略重力的影响, 另外, 也可以通过在坐标系 O-xyz 的运动中加入重力加速度的方法来计及重力的影响. 这样柔性梁的势能只由其弹性变形能构成，即

$$V =\frac{1}{2}EA\int_0^L\left(\frac{\partial u_{x1}}{\partial x}\right)^2\mathrm{d}x + \frac{1}{2}EI\int_0^L\left(\frac{\partial^2 u_y}{\partial x^2}\right)^2\mathrm{d}x+$$

$$\frac{1}{2}EI\int_0^L\Big(\frac{\partial^2 u_z}{\partial x^2}\Big)^2\mathrm{d}x. \tag{4}$$

式 (4) 的第 1 项为轴向拉伸势能，第 2 项、第 3 项对应于弯曲变形能， E 为弹性模量， EA 即为梁的抗拉压刚度， E 为截面惯性矩， EI 即为梁的抗弯刚度 (假定梁的抗弯刚度在 y 和 z 方向上是一样的，或梁为均匀圆截面), u_{x1} 如式 (8) 定义.

3.3 柔性梁动力学方程

本文采用假设模态法来描述柔性梁的变形，即

$$u_x = u_{x1} + u_{x2} + u_{x3}. \tag{5}$$

$$u_y(x,t) = \sum_{i=1}^{N}\phi_i(x)B_i(t). \tag{6}$$

$$u_z(x,t) = \sum_{i=1}^{N}\phi_i(x)C_i(t). \tag{7}$$

$$u_{x1}(x,t) = \sum_{i=1}^{N}\phi_{xi}(x)A_i(t). \tag{8}$$

$$u_{x2} = -\frac{1}{2}\int_0^x\Big(\frac{\partial u_y}{\partial \xi}\Big)^2\mathrm{d}\xi. \tag{9}$$

$$u_{x3} = -\frac{1}{2}\int_0^x\Big(\frac{\partial u_z}{\partial \xi}\Big)^2\mathrm{d}\xi. \tag{10}$$

u_{x1} 是轴向拉伸引起的 x 方向位移， u_{x2} 和 u_{x3} 分别是由于梁在 y 和 z 方向变形引起的 x 方向的位移， $\phi_{xi}(x), \phi_i(x)$, 分别为梁在轴向和横向的模态函数， $A_i(t), B_i(t), C_i(t)$ 为模态坐标， N 为模态截断数.

将式 (5) 、 (6) 和 (7) 表示的 u_x, u_y, u_z 代入式 (3) 和 (4), 从而动能 T 和势能 V 又可表示为

$$\begin{aligned}T =&\frac{1}{2}\rho A\int_0^L\Big\{\Big[v_{0x} - \omega_{0z}\sum_{i=1}^{N}\phi_i(x)B_i(t)+\\ &\omega_{oy}\sum_{i=1}^{N}\phi_i(x)C_i(t) + \sum_{i=1}^{N}\phi_{xi}(x)\dot{A}_i(t)-\\ &\underline{\sum_{i=1}^{N}\sum_{j=1}^{N}e_{ij}\dot{B}_iB_j} - \underline{\sum_{i=1}^{N}\sum_{j=1}^{N}e_{ij}\dot{C}_iC_j}\Big]^2+\\ &\Big[v_{0y} + \omega_{0z}\Big(x + \sum_{i=1}^{N}\phi_{xi}(x)A_i(t)-\\ &\underline{\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}e_{ij}B_iB_j} - \underline{\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}e_{ij}C_iC_j}\Big)-\\ &\omega_{ox}\sum_{i=1}^{N}\phi_{xi}(x)C_i(t) + \sum_{i=1}^{N}\phi_{xi}(x)\dot{B}_i(t)\Big]^2+\\ &\Big[v_{0z} - \omega_{0y}\Big(x + \sum_{i=1}^{N}\phi_{xi}(x)A_i(t)-\\ &\underline{\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}e_{ij}B_iB_j} - \underline{\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}e_{ij}C_iC_j}\Big)+\\ &\omega_{ox}\sum_{i=1}^{N}\phi_i(x)B_i(t) + \sum_{i=1}^{N}\phi_i(x)\dot{C}_i(t)\Big]^2\Big\}\mathrm{d}x.\end{aligned} \tag{11}$$

在式 (11) 及本文后述各表达式中下划线项都是由考虑横向弯曲所引起的纵向变形即式 (9) 和 (10) 所带来的增加项，而传统的动力学模型中不包含下划线项. 其中,

$$e_{ij}(x) = \int_0^x\phi_i'(\xi)\phi_j'(\xi)\mathrm{d}\xi, \tag{12}$$

$$\begin{aligned}V =&\frac{1}{2}EA\int_0^L\Big[\sum_{i=1}^{N}\phi_{xi}'(x)A_i(t)\Big]^2\mathrm{d}x+\\ &\frac{1}{2}EI\int_0^L\Big[\sum_{i=1}^{N}\phi_i''(x)B_i(t)\Big]^2\mathrm{d}x+\\ &\frac{1}{2}EI\int_0^L\Big[\sum_{i=1}^{N}\phi_i''(x)C_i(t)\Big]^2\mathrm{d}x.\end{aligned} \tag{13}$$

取广义坐标 $q = (A_1, A_2, \cdots, A_N, B_1, B_2, \cdots, B_N, C_1, C_2, \cdots, C_N)$ 将公式 (11) 和公式 (13) 表示的动能 T 和势能 V 代入第二类 Lagrange 方程

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} = -\frac{\partial V}{\partial q}. \tag{14}$$

求得柔性梁的三维振动方程为

$$\begin{bmatrix} M_{1,1} & \cdots & M_{1,N} & M_{1,N+1} & \cdots & M_{1,2N} & M_{1,2N+1} & \cdots & M_{1,3N} \\ M_{2,1} & \vdots & M_{2,N} & M_{2,N+1} & \vdots & M_{2,2N} & M_{2,2N+1} & \vdots & M_{2,3N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ M_{N,1} & \cdots & M_{N,N} & M_{N,N+1} & \cdots & M_{N,2N} & M_{N,2N+1} & \cdots & M_{N,3N} \\ M_{N+1,1} & \cdots & M_{N+1,N} & M_{N+1,N+1} & \cdots & M_{N+1,2N} & M_{N+1,2N+1} & \cdots & M_{N+1,3N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ M_{2N,1} & \cdots & M_{2N,N} & M_{2N,N+1} & \cdots & M_{2N,2N} & M_{2N,2N+1} & \cdots & M_{2N,3N} \\ M_{2N+1,1} & \cdots & M_{2N+1,N} & M_{2N+1,N+1} & \cdots & M_{2N+1,2N} & M_{2N+1,2N+1} & \cdots & M_{2N+1,3N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ M_{3N,1} & \cdots & M_{3N,N} & M_{3N,N+1} & \cdots & M_{3N,2N} & M_{3N,2N+1} & \cdots & M_{3N,3N} \end{bmatrix} \begin{bmatrix} \ddot{A}_1 \\ \ddot{A}_2 \\ \vdots \\ \ddot{A}_N \\ \ddot{B}_1 \\ \vdots \\ \ddot{B}_N \\ \ddot{C}_1 \\ \vdots \\ \ddot{C}_N \end{bmatrix} = \begin{bmatrix} Q_{A1} \\ Q_{A2} \\ \vdots \\ Q_{AN} \\ Q_{B1} \\ \vdots \\ Q_{BN} \\ Q_{C1} \\ \vdots \\ Q_{CN} \end{bmatrix} \tag{15}$$

式中,

$$M_{i,j} = m_{ij}, \quad M_{i,j+N} = -\sum_{k=1}^{N} F_{jki} B_k, \quad M_{i,j+2N} = -\sum_{k=1}^{N} F_{jki} C_k,$$

$$M_{i+N,j+N} = m_{ij} + \sum_{k=1}^{N}\sum_{l=1}^{N} E_{ikjl} B_k B_l, \quad M_{i+N,j} = -\sum_{k=1}^{N} F_{ikj} B_k,$$

$$M_{i+N,j+2N} = \sum_{k=1}^{N}\sum_{l=1}^{N} E_{ikjl} B_k C_l, \quad M_{i+2N,j+2N} = m_{ij} + \sum_{k=1}^{N}\sum_{l=1}^{N} E_{ikjl} C_k C_l,$$

$$M_{i+2N,j} = -\sum_{k=1}^{N} F_{ikj} C_k, \quad M_{i+2N,j+N} = \sum_{k=1}^{N}\sum_{l=1}^{N} E_{ikj} C_k B_l,$$

$$m_{ij} = \rho A \int_0^L \phi_i(x)\phi_j(x)\mathrm{d}x = \rho A \int_0^L \phi_{xi}(x)\phi_{xj}(x)\mathrm{d}x,$$

$$E_{ijkl} = \rho A \int_0^L \left(\int_0^x \phi_i'(\xi)\phi_j'(\xi)\mathrm{d}\xi \int_0^x \phi_k'(\xi)\phi_l'(\xi)\mathrm{d}\xi \right) \mathrm{d}x, \quad F_{ijk} = \rho A \int_0^L \left(\int_0^x \phi_i'(\xi)\phi_j'(\xi)\mathrm{d}\xi \right) \phi_{xk}(x)\mathrm{d}x,$$

$$Q_{Ai} = \sum_{j=1}^{N}\sum_{k=1}^{N} F_{jki} \left[\dot{B}_j \dot{B}_k + \dot{C}_j \dot{C}_k - \frac{1}{2}(\omega_{0z}^2 + \omega_{0y}^2)(B_j B_k + C_j C_k) \right] + (v_{oy}\omega_{0z} - v_{oz}\omega_{0y} - a_{0x}) G_{xi} +$$

$$\sum_{j=1}^{N} m_{xij} (2\omega_{0z}\dot{B}_j + \varepsilon_{0z} B_j - 2\omega_{0y}\dot{C}_j - \varepsilon_{0y} C_j - \omega_{0z}\omega_{0x} C_j - \omega_{0x}\omega_{0y} B_j) +$$

$$(\omega_{0z}^2 + \omega_{0y}^2) S_{xi} + \sum_{j=1}^{N} m_{ij} (\omega_{0z}^2 + \omega_{0y}^2) A_j - \sum_{j=1}^{N} k_{ij} A_j,$$

$$Q_{Bi} = \sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N} E_{ijkl} \left[\frac{1}{2}(\omega_{0z}^2 + \omega_{0y}^2)(B_j B_k B_l + B_j C_k C_l) - B_j \dot{B}_k \dot{B}_l - B_j \dot{C}_k \dot{C}_l \right] - \sum_{j=1}^{N} H_{ij} (\omega_{0z}^2 + \omega_{0y}^2) B_j +$$

$$\sum_{j=1}^{N}\sum_{k=1}^{N} f_{ijk} (\varepsilon_{0y} B_j C_k - \varepsilon_{0z} B_j B_k + \omega_{0z}\omega_{0x} B_j C_k + \omega_{0x}\omega_{0y} B_j B_k - 2\omega_{0z} B_j \dot{B}_k + 2\omega_{0y} B_j \dot{C}_k) +$$

$$\sum_{j=1}^{N}\sum_{k=1}^{N} f_{jki} \left[\frac{1}{2}(\omega_{0x}\omega_{0y} + \varepsilon_{0z})(B_j B_k + C_j C_k) + 2\omega_{0z}(\dot{B}_j B_k + \dot{C}_j C_k) \right] -$$

$$\sum_{j=1}^{N}\sum_{k=1}^{N}F_{ijk}(\omega_{0z}^2+\omega_{0y}^2)B_jA_k+\sum_{j=1}^{N}m_{ij}[2\omega_{0x}\dot{C}_j+(\varepsilon_{0x}-\omega_{0y}\omega_{0z})C_j+(\omega_{0z}^2+\omega_{0x}^2)B_j]-$$
$$\sum_{j=1}^{N}m_{xji}(2\omega_{0z}\dot{A}_j+\varepsilon_{0z}A_j+\omega_{0x}\omega_{0y}A_j)+\sum_{j=1}^{N}P_{ij}(a_{0x}+v_{0z}\omega_{0y}-v_{0y}\omega_{0z})B_j+(v_{oz}\omega_{0x}-v_{ox}\omega_{0z}-a_{0y})G_i-$$
$$(\varepsilon_{0z}+\omega_{0x}\omega_{0y})S_i-\sum_{j=1}^{N}K_{ij}B_j,$$

$$Q_{Ci}=\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N}E_{ijkl}\Big[\frac{1}{2}(\omega_{0z}^2+\omega_{0y}^2)(C_jC_kC_l+C_jB_kB_l)-C_j\dot{C}_k\dot{C}_l-C_j\dot{B}_k\dot{B}_l\Big]-$$
$$\sum_{j=1}^{N}H_{ij}(\omega_{0y}^2+\omega_{0z}^2)C_j+$$
$$\sum_{j=1}^{N}\sum_{k=1}^{N}f_{ijk}(\varepsilon_{0y}C_jC_k-\varepsilon_{0z}C_jB_k+\omega_{0z}\omega_{0x}C_jC_k+\omega_{0x}\omega_{0y}C_jB_k+2\omega_{0y}C_j\dot{C}_k-2\omega_{0z}C_j\dot{B}_k)+$$
$$\sum_{j=1}^{N}\sum_{k=1}^{N}f_{jki}\Big[\frac{1}{2}(\omega_{0z}\omega_{0x}-\varepsilon_{0y})(B_jB_k+C_jC_k)-2\omega_{0y}(\dot{B}_jB_k+\dot{C}_jC_k)\Big]-$$
$$\sum_{j=1}^{N}\sum_{k=1}^{N}F_{ijk}(\omega_{0z}^2+\omega_{0y}^2)C_jA_k+\sum_{j=1}^{N}m_{ij}[-2\omega_{0x}\dot{B}_j-(\varepsilon_{0x}+\omega_{0y}\omega_{0z})B_j+(\omega_{0y}^2+\omega_{0x}^2)C_j]+$$
$$\sum_{j=1}^{N}m_{xji}(2\omega_{0y}\dot{A}_j+\varepsilon_{0y}A_j-\omega_{0z}\omega_{0x}A_j)+\sum_{j=1}^{N}P_{ij}(a_{0x}+v_{0z}\omega_{0y}-v_{0y}\omega_{0z})C_j+$$
$$(v_{ox}\omega_{0y}-v_{oy}\omega_{0x}-a_{0z})G_i+(\varepsilon_{0y}-\omega_{0z}\omega_{0x})S_i-\sum_{j=1}^{N}K_{ij}C_j,$$

$$f_{ijk}=\rho A\int_0^L\left(\int_0^x\phi_i'(\xi)\phi_j'(\xi)\mathrm{d}\xi\right)\phi_k(x)\mathrm{d}x,\quad H_{ij}=\rho A\int_0^L\left(\int_0^x\phi_i'(\xi)\phi_j'(\xi)\mathrm{d}\xi\right)x\mathrm{d}x,$$
$$P_{ij}=\rho A\int_0^L\int_0^x\phi_i'(\xi)\phi_j'(\xi)\mathrm{d}\xi\mathrm{d}x,\quad m_{xji}=\rho A\int_0^L\phi_{xj}(x)\phi_i(x)\mathrm{d}x,\quad m_{xij}=\rho A\int_0^L\phi_{xi}(x)\phi_j(x)\mathrm{d}x,$$
$$S_{xi}=\rho A\int_0^L x\cdot\phi_{xi}(x)\mathrm{d}x,\quad S_i=\rho A\int_0^L x\cdot\phi_i(x)\mathrm{d}x,\quad G_{xi}=\rho A\int_0^L\phi_{xi}\mathrm{d}x,$$
$$G_i=\rho A\int_0^L\phi_i\mathrm{d}x,\quad k_{ij}=EA\int_0^L\phi_{xi}'(x)\phi_{xj}'(x)\mathrm{d}x,\quad K_{ij}=EI\int_0^L\phi_i''(x)\phi_j''(x)\mathrm{d}x.$$

从上面各式中可以看出，我们所得的动力学模型与传统模型 (不含下划线部分) 有较大的差别，下面将通过数值仿真例子来说明这一点，并且发现，在有些情况下，两种模型将导致完全相反的结果.

4 仿真实例分析

下面对一空间运动基础上悬臂梁的动力学进行仿真. 设均质圆形截面梁长 $L=10.0\,\mathrm{m}$, 横截面积 $A=0.0004\,\mathrm{m}^2$, 截面惯性矩 $I=2.0\times10^{-7}\,\mathrm{m}^4$, 总质量 $M=12\,\mathrm{kg}$, 弹性模量 $E=7.0\times10^{10}\,\mathrm{N/m}^2$, T 为加速时间， $T=15\,\mathrm{s}$, 仿真总时间为 $30\,\mathrm{s}$, 基础运动规律为 [1]

$$\begin{aligned}\boldsymbol{v}_0=&v_{0x}\boldsymbol{i}+v_{0y}\boldsymbol{j}+v_{0z}\boldsymbol{k},\\ v_{0x}=&l(1+\bar{C}_2)\bar{S}_3\dot{\theta}_1,\quad v_{0y}=l\dot{\theta}_2,\\ v_{0z}=&-l(1+\bar{C}_2)\bar{C}_3\dot{\theta}_1\end{aligned}$$

和

$$\boldsymbol{\omega}_0=\omega_{0x}\boldsymbol{i}+\omega_{0y}\boldsymbol{j}+\omega_{0z}\boldsymbol{k},$$

$$\omega_{0x} = \dot{\theta}_1 \bar{S}_2 \bar{C}_3 - \dot{\theta}_2 \bar{S}_3,$$
$$\omega_{0y} = \dot{\theta}_1 \bar{C}_2 + \dot{\theta}_3,$$
$$\omega_{0z} = \dot{\theta}_1 \bar{S}_2 \bar{S}_3 + \dot{\theta}_2 \bar{C}_3.$$

在这里 $\bar{C}_i \triangleq \cos\theta_i$, $\bar{S}_i \triangleq \sin\theta_i$ $(i = 1, 2, 3)$.

对两种运动情形进行仿真结果如下.

第一种运动情形 (取 $l = 8$ m)

$$\theta_1(t) = \begin{cases} \pi - (\pi/2T) \cdot [t - (T/2\pi)\sin(2\pi t/T)], \\ \qquad\qquad 0 \leqslant t \leqslant T; \\ \pi/2, \qquad\qquad t > T. \end{cases}$$

$$\theta_2(t) = \begin{cases} \pi - (3\pi/4T) \cdot [t - (T/2\pi)\sin(2\pi t/T)], \\ \qquad\qquad 0 \leqslant t \leqslant T; \\ \pi/4 \qquad\qquad t > T. \end{cases}$$

$$\theta_3(t) = \begin{cases} \pi - (\pi/T) \cdot [t - (T/2\pi)\sin(2\pi t/T)], \\ \qquad\qquad 0 \leqslant t \leqslant T; \\ 0, \qquad\qquad t > T. \end{cases}$$

第二种运动情形 (取 $l = 1$ m)

$$\theta_1(t) = \begin{cases} \pi - (\pi/2T) \cdot [t - (T/2\pi)\sin(2\pi t/T)], \\ \qquad\qquad 0 \leqslant t \leqslant T; \\ \pi/2, \qquad\qquad t > T. \end{cases}$$

$$\theta_2(t) = \begin{cases} 6/T \cdot [t^2/2 + (T/2\pi)^2 \cdot (\cos(2\pi t/T) - 1)], \\ \qquad\qquad 0 \leqslant t \leqslant T; \\ 6t - 45, \qquad\qquad t > T. \end{cases}$$

$$\theta_3(t) = 0, \qquad t > 0.$$

仿真结果如图 2~10 所示, 在图中实线代表本文情形, 点线代表传统模型. 图 2~4 代表第一种运动情形, 图 5~10 代表第二种运动情形.

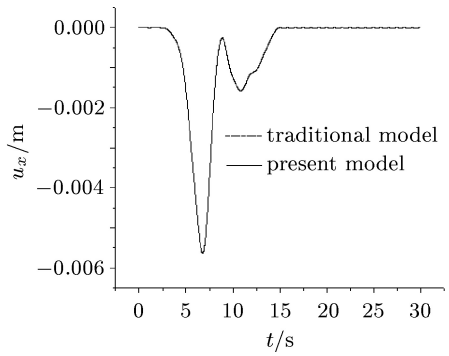


图 2 末端位移在 x 方向变化曲线

Fig.2 Tip deflection in x direction

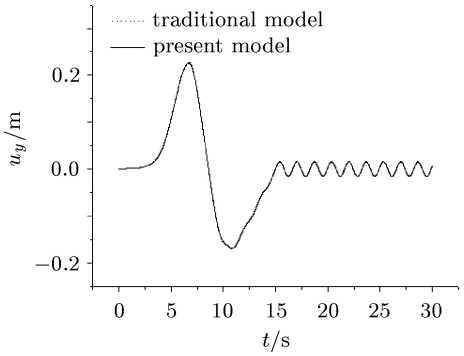


图 3 末端位移在 y 方向变化曲线

Fig.3 Tip deflection in y direction

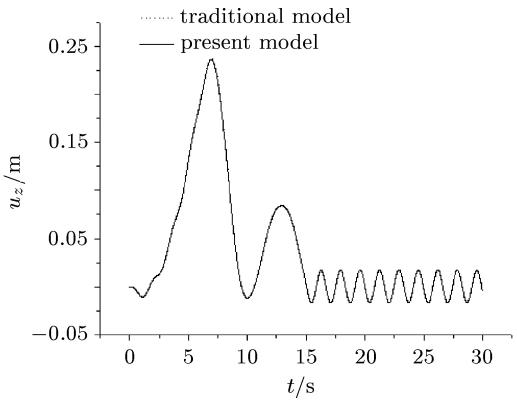


图 4 末端位移在 z 方向变化曲线

Fig.4 Tip deflection in z direction

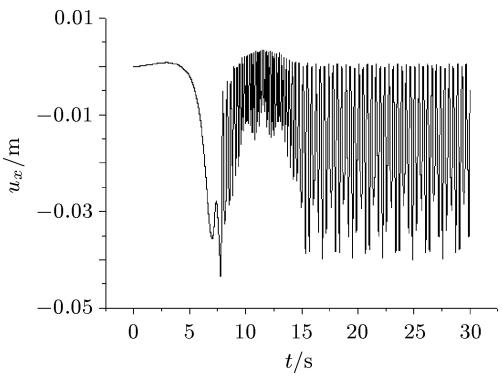


图 5 末端位移在 x 方向变化曲线 (本文模型)

Fig.5 Tip deflection in x direction (present model)

从上述图 2~4 (第一运动情形) 看出, 当基础的运动处在低角速度时, 本文结果与传统结果非常接近, 以致于在图中难以分辨, 系统均处在稳定状态. 而在高角速度时 (第二运动情形), 从图 5~10 中看出, 本文结果 (图 5~7) 是稳定的, 而传统模型 (图 8~10) 发散, 这说明本文模型考虑柔性梁的横向弯

曲所带来的纵向位移将使系统产生动力刚化效应，在柔性梁作高角速度大位移运动时，纵横变形之间的耦合将对柔性梁的动力学性态具有很大的影响. 传统的空间运动体上的梁振动模型具有局限性，当旋转角速度较大时，模型的精度就难以保证，角速

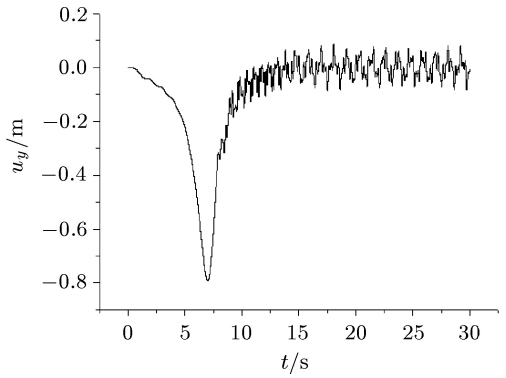


图 6　末端位移在 y 方向变化曲线 (本文模型)

Fig.6　Tip deflection in y direction (present model)

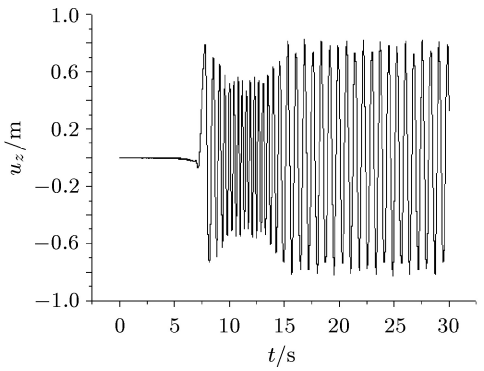


图 7　末端位移在 z 方向变化曲线 (本文模型)

Fig.7　Tip deflection in z direction (present model)

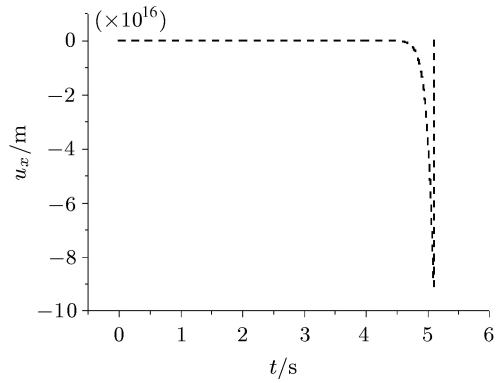


图 8　末端位移在 x 方向变化曲线 (传统模型)

Fig.8　Tip deflection in x direction (traditional model)

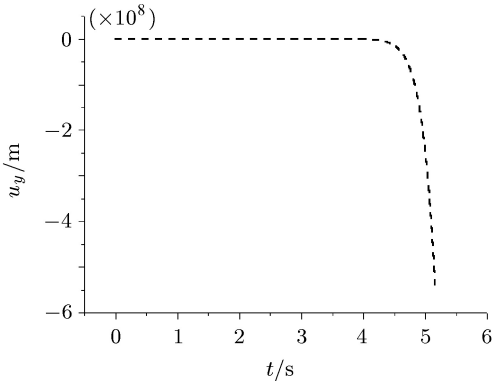


图 9　末端位移在 y 方向变化曲线 (传统模型)

Fig.9　Tip deflection in y direction (traditional model)

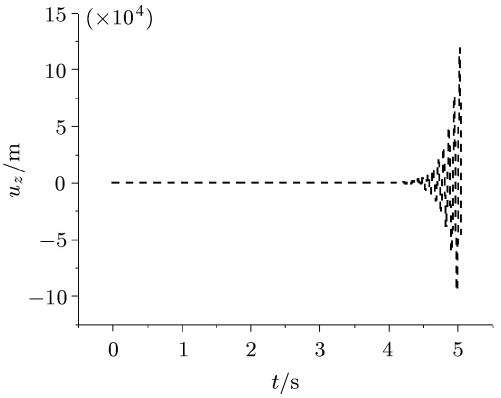


图 10　末端位移在 z 方向变化曲线 (传统模型)

Fig.10　Tip deflection in z direction (traditional model)

度一旦大于梁在结构动力学下的一阶固有频率，动力学方程出现负刚度，变形趋大并发散，传统模型只保证在低速下有较高精度，高速旋转下梁的动力刚化效应不容忽视. 通过设定空间运动体的运动模式，本文动力学模型可以退化到中心刚体-外梁、中心刚体-内梁和中心刚体-纵向梁等情形，但显然本文结论更具一般性. 在文章中给出了空间运动体上梁的三维动力学方程的显式详细的式子，读者可以用来进行不同层次的运动基上梁的动力学分析. 另外本文所用方法是程式化的推导方法，可以用相同的方法将动力刚化项程式化地引入柔性多体系统动力学方程，作者将另文介绍这项颇具意义和挑战性的工作.

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